The Study on Fuzzy Comprehensive Evaluation Method Using in Underground Space Resources Quality Evaluation

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ABSTRACT

In order to master the situation of underground space resources quality and avoid blind development, we must evaluate the quality of underground space resources. Based on the analysis of factors which affect the quality of underground space resources and by using multi-level fuzzy comprehensive evaluation model, this article assess the quality of urban underground space. In the process of modelling, AHP is used to determine weight, and main factors used to synthesize result, so we gained the grade of urban underground space resources quality. The evaluation of underground space resources can guide developer to orderly exploit and utilize underground space.

1. URBAN GROUND SPACE RESOURCES QUALITY

The quality of underground space usually refers to development and utilization of urban underground space. It can be express by the relative scores or grade after integrated evaluation. Quality assessment of the underground space can provide guidance for the development of underground space, in order to avoid the development of blindness in decision making. For urban underground space in terms of quality, it involved many factors. In these factors, the first is their respective attributes, degree of importance and comparability. The Second is prodigious imprecise and subjective experiential during the measurement and assessment. Therefore, the evaluation and selection of the best quality of the underground space is a complex system of multi-level fuzzy environment and multi-attribute decision-making. Just the fuzzy comprehensive evaluation with a variety of attributes is adopt to it which affected by a number of factors and these attributes or ambiguous nature of the evaluation factors, Therefore, in this paper we use fuzzy comprehensive evaluation of the quality of urban underground space for assessment and classification.

2. ESTABLISHING QUALITY ASSESSMENT MODEL BY FUZZY MATHEMATICS

According to the fuzzy math theory, the first assumption evaluation object set \( X = \{x_1, x_2, \ldots \} \), quality factors set \( U = \{u_1, u_2, \ldots, u_n\} \), intituled \( u_i \) is the quality factor, evaluation grades Set \( V = \{v_1, v_2, \ldots, v_m\} \), Usually, have uniform standards for each quality factor, mostly denoted by digital form. Evaluation criteria can be given grades of quality indicators or can be grading scope. As shown in table 1, in which the scope for grades indicated in parentheses to the standards set. The bracket in the table denoted criterion of grade scope, \( a_{ij} \) denoted Num.i quality factor is belong to class j indicator which is \( \geq a_{ij} \) or \( \leq a_{ij} \). Because of the different quality factors of evaluation object \( x_i \) is usually belong to
different grades. So we must use a comprehensive evaluation method to synthesis different quality factor for drawing a comprehensive evaluation result.

Table 1. Grade table of quality factors.

<table>
<thead>
<tr>
<th></th>
<th>(v_1)</th>
<th>(v_2)</th>
<th>(\ldots)</th>
<th>(v_{m-1})</th>
<th>(v_m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_1)</td>
<td>(a_{11})</td>
<td>(a_{12})</td>
<td>(a_{1,m-1})</td>
<td>(a_{1m})</td>
<td></td>
</tr>
<tr>
<td>(u_2)</td>
<td>(a_{21})</td>
<td>(a_{22})</td>
<td>(a_{2,m-1})</td>
<td>(a_{2m})</td>
<td></td>
</tr>
<tr>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td>(\ldots)</td>
<td></td>
</tr>
<tr>
<td>(u_{m-1})</td>
<td>(a_{m-1,1})</td>
<td>(a_{m-1,2})</td>
<td>(a_{m-1,m-1})</td>
<td>(a_{m-1,m})</td>
<td></td>
</tr>
<tr>
<td>(u_m)</td>
<td>(a_{m,1})</td>
<td>(a_{m,2})</td>
<td>(a_{m,m-1})</td>
<td>(a_{mm})</td>
<td></td>
</tr>
</tbody>
</table>

2.1 Establishment of membership function

By using fuzzy comprehensive evaluation firstly we must establish the membership functions and fuzzy relationship matrix. The form of indicators of \(a_{ij}\) was given in the table, if \(i \geq a_{ij}\), obviously, \(a_{i1} \geq a_{i2} \geq \ldots \geq a_{im}\), here we stipulate the membership function as follows:

\[
\mu_{i1}(x) = \begin{cases} 
1, & x \geq a_{i1} \\
\left(\frac{a_{i2} - x}{a_{i2} - a_{i1}}\right)^{\delta}, & a_{i2} \leq x < a_{i1} \\
0, & x < a_{i2} 
\end{cases}
\]

\[
\mu_{ij}(x) = \begin{cases} 
1, & x \geq a_{ij} \\
\left(\frac{a_{i,j+1} - x}{a_{i,j+1} - a_{ij}}\right)^{\delta}, & a_{i,j+1} \leq x < a_{ij} \\
0, & x < a_{i,j+1} 
\end{cases}
\]

\[
\mu_{im}(x) = \begin{cases} 
1, & x \geq a_{im} \\
\left(\frac{x}{a_{im}}\right)^{\delta}, & x \leq a_{im} \\
0, & x < a_{im} 
\end{cases}
\]

Among them, when \(u_{ik} = 1\), \(\delta = \begin{cases} 
2, & j < k \\
1, & j > k 
\end{cases}\)

if \(i \geq a_{ij}\), obviously, \(a_{i1} \leq a_{i2} \leq \ldots \leq a_{im}\), here we stipulate the membership function as follows:
\[ \mu_i(x) = \begin{cases} 1, & x \leq a_{i_1} \\ \frac{a_{i_2} - x}{a_{i_2} - a_{i_1}}, & a_{i_1} < x \leq a_{i_2} \\ 0, & x > a_{i_2} \end{cases} \]

\[ \mu_j(x) = \begin{cases} \frac{x}{a_{j,p}}, & x \leq a_{j,p} \\ 1, & a_{j,p} < x \leq a_{j,p+1} \\ \frac{a_{j,p+1} - x}{a_{j,p+1} - a_{j,p}}, & a_{j,p} < x \leq a_{j,p+1} \\ 0, & x > a_{j,p+1} \end{cases} \]

\[ \mu_{i,n}(x) = \begin{cases} \frac{x}{a_{i,n-1}}, & x \leq a_{i,n-1} \\ 1, & a_{i,n-1} < x \leq a_{i,n} \\ \frac{a_{i,n} - x}{a_{i,n} - x}, & x > a_{i,n} \end{cases} \]

\[ \delta \] is the same as above.

If \((a_{j-1} - a_j)\), here we stipulate the membership function as follows:

\[ \mu_{i,j}(x) = \begin{cases} 1, & x \text{ between } a_{i,j-1} \text{ and } a_{i,j} \\ \frac{a_{i,j} - x}{a_{i,j} - a_{i,j-1}}, & x \text{ between } a_{i,j-1} \text{ and } a_{i,j} \\ 0, & x \text{ between } a_{i,j-1} \text{ and } a_{i,j} \end{cases} \]

\[ \mu_{i,j}(x) = \begin{cases} 0, & x \text{ between } a_{i,j-2} \text{ and } a_{i,j-1} \\ \frac{x - a_{i,j-2}}{a_{i,j-2} - a_{i,j-3}}, & x \text{ between } a_{i,j-2} \text{ and } a_{i,j-1} \\ 1, & x \text{ between } a_{i,j-1} \text{ and } a_{i,j} \\ \frac{a_{i,j+1} - x}{a_{i,j+1} - a_{i,j}}, & x \text{ between } a_{i,j} \text{ and } a_{i,j+1} \\ 0, & x \text{ between } a_{i,j} \text{ and } a_{i,j+1} \end{cases} \]

\[ \mu_{i,n}(x) = \begin{cases} 0, & x \text{ between } a_{i,n-2} \text{ and } a_{i,n-1} \\ \frac{x - a_{i,n-2}}{a_{i,n-2} - a_{i,n-3}}, & x \text{ between } a_{i,n-2} \text{ and } a_{i,n-1} \\ 1, & x \text{ between } a_{i,n-1} \text{ and } a_{i,n} \end{cases} \]

2.2 Establishment of fuzzy relationship matrix

Each evaluation object \(x_k\), by sampling inspection, for each quality factor \(u_i\), can get a measured \(y_i\), and then get a measured index vectors \(Y_k = (y_1, y_2, ..., y_n)\) which corresponding to the \(x_k\).
Now $u_i(y)$ can be denote degree belonging to $v_i$ for $x_k$ relative to the factor $u_i$, can get

$$R_k = \begin{bmatrix}
\mu_{i1}(y_1) & \mu_{i2}(y_1) & \cdots & \mu_{in}(y_1) \\
\mu_{i1}(y_2) & \mu_{i2}(y_2) & \cdots & \mu_{in}(y_2) \\
\vdots & \vdots & \ddots & \vdots \\
\mu_{i1}(y_n) & \mu_{i2}(y_n) & \cdots & \mu_{in}(y_n)
\end{bmatrix}_{m \times n}$$

Now $R_k$ is a fuzzy matrix which is relationship between $U$ and $V$ about $x_k$. If give it a weight $A = (a_1, a_2, \ldots, a_n)$ $\sum_{i=1}^{n} a_i = 1$

Calculate $B_k = A \circ R_k = (b_1, b_2, \ldots, b_m)$

According to the largest subjection principle, if $b_{j0} = \max_{1 \leq j \leq m} b_j$, then assess $x_k$ belonging to $v_{j0}$ degree.

3. ESTABLISHING ASSESSMENT FACTOR SET OF UNDERGROUND SPACE

The quality of urban underground space was restricted by many factors. We selected the following indicators as a basis and reference in the assessment process. First is the regional tectonic of assess unit. Second is nature of the soil (or rock) of the underground space for the development. Third is the hydrogeological. In hydrogeological we select the second level indicators including depth of ground water, single Well outflow and ground water corrosive to concrete and so on. Now we take the rock indicators to illustrate the process of construction membership function and evaluation calculation.

<table>
<thead>
<tr>
<th>Name</th>
<th>Index</th>
<th>Evaluation grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>hard rock (&gt;60 MPa)</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>less hard rock (60 – 30 MPa)</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>less soft rock (30 – 15 MPa)</td>
<td>III</td>
</tr>
<tr>
<td></td>
<td>Soft rock (&lt;15 MPa)</td>
<td>IV</td>
</tr>
</tbody>
</table>

4. THE MEMBERSHIP FUNCTION OF ROCK

According to set of evaluation factors, establishment their membership function. Take rock as an example, the compressive strength of a single piece of rock as a reference standard to establish membership function as follows:

1: $y = \begin{cases} 
1 \quad x \geq 60 \\
1 - 2 \left( \frac{x - 60}{60 - 30} \right)^2 \quad 45 \leq x \leq 60 \\
2 \left( \frac{x - 30}{60 - 30} \right)^2 \quad 30 \leq x \leq 45 \\
0 \quad x \leq 30
\end{cases}$

II: $y = \begin{cases} 
2 \left( \frac{x - 60}{60 - 30} \right)^2 \quad 45 \leq x \leq 60 \\
1 - 2 \left( \frac{x - 30}{60 - 30} \right)^2 \quad 30 \leq x \leq 45 \\
1 - 2 \left( \frac{x - 30}{30 - 20} \right)^2 \quad 25 \leq x \leq 30 \\
2 \left( \frac{x - 20}{30 - 20} \right)^2 \quad 20 \leq x \leq 25 \\
0 \quad x < 20, x > 60
\end{cases}$
To other indicators we can also be set up the membership function by using the same way. Because of limited space is no longer description other here.

5. APPLICATION EXAMPLE

Using these factors set and the establishment of the membership function, we can use multi-level fuzzy assessment to evaluate quality of the urban underground space. multi-level fuzzy evaluation course map as shown in Fig 1. The parameter values is shown in Table 3.

Table 3. The parameter values of instance.

<table>
<thead>
<tr>
<th>first level evaluation factors</th>
<th>second level evaluation factors</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tectonic (the number of faultage) $U_i$</td>
<td>soil bearing capacity</td>
<td>2</td>
<td>bar</td>
</tr>
<tr>
<td>the quality of underground space $U_z$</td>
<td>rock compressive strength</td>
<td>100</td>
<td>KPa</td>
</tr>
<tr>
<td>hydrogeological $U_u$</td>
<td>depth of ground water</td>
<td>45</td>
<td>MPa</td>
</tr>
<tr>
<td></td>
<td>single well outflow</td>
<td>2</td>
<td>m</td>
</tr>
<tr>
<td></td>
<td>ground water corrosive(ph value)</td>
<td>100</td>
<td>M³/D</td>
</tr>
</tbody>
</table>

Firstly, we assess the secondary indicators, as an example of hydrogeology, according the membership functions, we can get judge result of the single factor :

Judge result of the single factor of depth of ground water : $(0, 1, 0, 0)$
Judge result of the single factor of single well outflow : $(1, 0, 0, 0)$
Judge result of the single factor of ground water corrosive : $(0, 0, 0, 1)$

Now $R_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, fuzzy weight factor $A_i = (0.3, 0.5, 0.2)$, using main factors prominence arithmetic operators $b_j = \sqrt[n]{\sum_{i=1}^{n} (a_i r_{ij})}$, $j = 1, 2, \cdots, m$, using fuzzy evaluation
Get the result of $B_3$, it shows that the groundwater of the overall situation belonging to fuzzy subset of the various grades of membership degree. Its value will participate in the urban underground space first level evaluation. The same as groundwater, Rock structure $B_2 = (0.145, 0.145, 0.71, 0)$ Tectonic $B_1 = (0.5, 0.5, 0, 0)$ So we can use those data to evaluate first fuzzy comprehensive assessment, as follows

\[
A \circ R = A \circ B_2 = (0.284, 0.3233, 0.3927) \circ (0.145, 0.145, 0.71, 0) \circ (0.5, 0.3, 0, 0.2) = 0.1964, 0.142, 0.230, 0.0785
\]

using the main factors prominence arithmetic operators to synthesis the result of evaluation and using the method of maximum degree of membership to analysis the assessment. We can get the result of underground space resources quality. In the example the evaluation result is class III. The calculation weight factors shown in Table 4. In the same way, we can evaluate all area of underground space resource quality and their distribution and a map of quality distribution of underground space can be drawn (Fig. 2). It can guide the development and utilization of urban underground space.

### Table 4. Result of second level evaluation and data of instance.

<table>
<thead>
<tr>
<th>First Level evaluation factors</th>
<th>Second Level evaluation factors</th>
<th>Vector single factor evaluation results</th>
<th>Weight of second level</th>
<th>Weight of first level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tectonic $u_1$ (the number of faultage)</td>
<td></td>
<td>0.5 0.5 0 0</td>
<td>0.2840</td>
<td></td>
</tr>
<tr>
<td>The quality of underground space</td>
<td>Rock structure $u_2$</td>
<td>soil bearing capacity</td>
<td>0 0 1 0</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rock compressive strength</td>
<td>0.5 0.5 0 0</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>depth of ground water single Well outflow</td>
<td>0 1 0 0</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Hydrogeological $u_3$</td>
<td>ground water corrosive</td>
<td>1 0 0 0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0 0 0 1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The weight coefficient of rock structure is according to rocks and soil and their proportion in the depth in the area of assessment. Other Weights come from expert advice, using the analytic hierarchy process analysis.
6. CONCLUSION

Fuzzy mathematical model assesses the quality of underground space not only an objective response of the true quality of underground space and make quantitative characterization. The calculation process clear, simple judgment, but who can tell the difference between the block and the block and their quality level. This method has certain practical and operable. It can be applied to the underground space quality assessment.

REFERENCES

Yuan Shenhuan, Li Hongxing, 1986. The mathematical model of rating the quality. The practice and understanding of math. 1986, No.2