Stability of Multi-Jointed Roof Rock Beams in Large Underground Openings

Pavlos P. Nomikos¹, Paraskevi Yiouta-Mitra¹, Alexandros I. Sofianos¹

¹ National Technical University, School of Mining and Metallurgy, Athens, Greece

ABSTRACT

The stability of long shallow underground openings depends mainly on the bearing capacity of the lowest roof rock strata. Such strata are usually jointed and may be considered as discontinuous rock beam or plate structures. Their mechanical behaviour has been studied in the past by considering the mechanical analogue of a three-jointed voussoir beam. Provided that the stiffness of the rock joints is high enough, existing analytical solutions may be used for the calculation of the mechanical parameters of the problem. However, roof rock beams usually are crossed with more than three joints which have further low stiffness values. For such beams no analytical solution exists and their mechanical behaviour has been examined numerically. In this paper, the mechanical behaviour of such rock beams is described and simple methodologies for the approximation of their mechanical parameters are presented. Further, their stability against snap-through failure mechanism is numerically examined with a distinct element code.

1. INTRODUCTION

Arching phenomena in jointed roof beds have been studied initially by Fayol (1885), who noticed experimentally that the lowest stratum of the roof was not loaded by the upper ones. Significant increase in the unloaded volume is observed with decreasing of the interbed friction angle (Sofianos et. al., 2000), rendering, in most cases, the single unloaded stratum assumption as relevant. Evans (1941) studied the behaviour of such a single bed roof and provided graphs, tables and algorithms for the prediction of its stability. Wright (1974) created a design procedure for the control of layered, bolted or unbolted, underground, undermined roofs, based on finite element and laboratory experiment results. Sterling (1980) captured the salient features of the work of Evans and other researchers and by experimenting on his apparatus provided a coherent picture of the deformation and failure modes of roof rock.

Based on Evans’s assumptions and finite element continuous analysis computer output data obtained by Wright, Sofianos (1996) provided an analytical solution for the response of an idealised symmetric cracked roof beam of hard rock for three modes of failure, i.e. snap through, crushing and sliding, which modes were included in a single non dimensional chart. There, among the assumptions made, the joints were assumed to exist at the abutments and midspan only, and to be very stiff. However, many of the roofs of underground excavations may contain distributed cracks all over their length, which may be softer than the surrounding rock. This was noted by Brady and Brown (1985), who suggested an explicit account to be taken of the presence and compressibility of cross joints in estimating the elastic modulus. Further, any abutment joint infillings increase the compliance of the beam and result in lowering the lever arm of the resultant thrust force. Sofianos et. al. (2000) and Nomikos et. al. (2002) investigated on the behaviour of such roofs and on the divergence of their response from the pertinent analytic formulae for the idealised structure.
2. THREE STIFF JOINT VOUSSOIR BEAM

In Figure 1, a three joint voussoir beam model is drawn; pertinent notation for its geometry and loading may be seen there.

\[
Q_n = k_q \cdot \gamma \cdot \frac{S}{E}, s_n = \frac{s}{t}, s_z = \frac{s}{z_o}, \delta_z = \frac{\delta}{z_o}, z_{on} = \frac{z}{t} + \delta, z_{on} = 1 - \frac{2}{3} \cdot n, n = \frac{h}{t}
\]

(1)

where, \(E\) is the intact rock modulus of deformability; \(\gamma\) the unit weight of the rock; and \(k_q\) a surcharge factor which for a beam loaded with its self weight only has the value 1.

For small deflections of the beam, the deflection and the extreme strain have been evaluated for the three stiff joint symmetric voussoir rock beam of Fig. 1 (Sofianos and Kapenis, 1998), to be:

\[
\delta_z = \frac{Q_n \cdot s_z}{16} \cdot \left(s_z^2 + k_1\right) \approx \frac{Q_n \cdot s_z^3}{16}, \varepsilon_x = \frac{\sigma_x}{E} = \frac{Q_n \cdot s_z}{4 \cdot n}
\]

(3)

where, \(k_1\) a parameter reflecting the actual shape of the thrust line within the beam.

If larger deflections are anticipated, \(\delta_z = \delta_{z_0}\) may be evaluated from Eqs. (4) through (6):

\[
\delta_z = 1 - \cos \omega / \sqrt{3 - \sin \omega}
\]

(4)

\[
\omega = \frac{1}{3} \arctan \left(\frac{\sqrt{\frac{1}{27} - \delta_{z_0}^2}}{\delta_{z_0}}\right)
\]

(5)

\[
\omega \in \left[\frac{\pi}{6}, 0\right], \delta_{z_0} \in \left[0, \frac{1}{\sqrt{27}}\right], \delta_z \in \left[0, \frac{\sqrt{3} - 1}{\sqrt{3}}\right]
\]

(6)

where \(\chi\) is the mean thickness of the compression arch.

In the above formulae there still remain three parameters of indeterminacy, i.e. \(k_1, \chi\) and \(n\), to be evaluated. The former is defined by Eq. (7):

\[
k_1 = \left(\frac{L}{s} - 1\right) \cdot s_z^2
\]

(7)

where, \(L\) is the length of the thrust line; the value of the parameter is of the order of 8/3 and for values of \(s_z\) larger than 15 its contribution becomes insignificant. Statistical interpretation of a large number of beams investigated numerically for common values of rock unit weight and deformability yielded
3. MULTI-JOINTED BEAMS WITH SMALL DEFLECTIONS

The three stiff joint symmetric beam model allows for the analytical evaluation of its mechanical response. However, increase in the frequency and compliance of the joints crossing an underground roof beam, modify its response. Nomikos et al. (2002) examined the structural response of multi-jointed roof beams with various span to thickness ratios, that comprise a wide range of practical concern, with the distinct element code UDEC (Itasca, 1999). These beam models, tested for small deflections, indicate an increase in the deflection and a decrease in the extreme strain especially at the abutment, with the increase in the joint frequency and compliance.

The contact length at the abutment increases due to the increase in the joint compliance and frequency, whereas at midspan this parameter is of minor importance. This increase in the contact length causes a reduction in the level arm. The increase in the deflection and contact length due to the joint compliance starts to be significant for low stiffness values.

Results obtained by Nomikos et al. (2002) indicate a strong correlation of the normalized contact length at the abutment (assuming a triangular stress distribution there) with the ratio of rock beam compliance to the joint compliance as may be observed in Fig. 2a. Rock beam compliance depends on the equivalent modulus of deformation of the rock beam consisting of intact rock blocks and joints. This may be considered to be the deformation modulus of the rock/joint system along the span of the beam which is calculated from Eq. (10):

\[
\frac{1}{E'} = \frac{1}{E} + \frac{1}{k_nS_j}
\]  

where \( E' \) is the deformation modulus of the rock/joint system; \( k_n \) is the normal stiffness of the discontinuities; and \( s_j \) the spacing of the discontinuities along the beam.

Further, the ratio of the normalized contact length at the abutment to the normalized contact length at midspan (for a triangular stress distribution) may be also correlated with the ratio of rock beam compliance to the joint compliance and the slenderness of the beam, as may be observed in Fig. 2b.

Graphs of Fig. 2 may be used for preliminary design purposes of multiple jointed roof beams pertinent to those analyzed by Nomikos et al. (2002). For this purpose the value of normalized contact length at the abutment \( n_a \) may be obtained from the graph of Fig. 2a. The normalized contact length at midspan \( n_m \) may be evaluated from the ratio of \( n_a \) to \( n_m \) obtained from Fig. 2b. Having estimated the values of \( n_a \) and \( n_m \), the lever arm \( z_0 \) before deflection of the beam may be evaluated from Eq. (11):

\[
z_0 = t \left[ 1 - \frac{(n_a + n_m)}{3} \right]
\]
Deflection of the beam at midspan may be obtained as an approximation for the graph of Fig. 3, where a correlation of $\delta$ with the factor $Q'_n s z^3 \left( s z^2 + k_1 \right) z_0 / 16$ is shown. $Q'_n$ is the normalized load of the beam evaluated from Eq. (1) substituting $E$ with $E'$ and assigning a value for $k_1 = 1$ (i.e. the beam is loaded by its own weight only).

![Graph of $\delta$ as a function of $Q'_n s z^3 \left( s z^2 + k_1 \right) z_0 / 16$.](image)

The lever arm $z$ after deflection of the beam may be evaluated from Eq. (2) and the magnitude of the horizontal thrust at the abutment is calculated by equating the resisting moment due to the horizontal thrust $T$ at the abutment or at midspan with the overturning couple:

$$T = \frac{\gamma \cdot s \cdot t}{8 \cdot z} \quad (12)$$

Finally, the axial strains at the abutment and at midspan may be evaluated by Eq. (13)

$$\varepsilon_{ax} = 2T / (n_m Et) \quad , \quad \varepsilon_{mx} = 2T / (n_m Et) \quad (13)$$

Thus the beam may be checked against shear failure at the abutments or crushing of the rock at the abutments and at midspan. There is still one more failure mode to be examined that is investigated in the following.

4. STATE OF BUCKLING

4.2 Numerical model

The mode of failure which is usually referred in rock mechanics literature as buckling or snap through mechanism is a limiting structural stability condition of the rock beam at which any farther deflection provides less beam resistance. For a beam with given span there will be a minimum thickness, further of which snap through occurs. This minimum thickness values for beams of various spans, moduli of elasticity, joint frequencies and compliances is computed herein by UDEC (Itasca, 1999), which is a two dimensional computer code based on the Distinct Element Method.

Beams of four different spans, i.e. $s = 5, 10, 20$ and $40 \text{ m}$, that comprise a wide range of practical concern, are investigated. The model consists of three blocks, one deformable in the centre, which simulates the half beam and two rigid, one at each side of the half beam. The left one simulates the abutment and the right one imposes the appropriate boundary conditions at the midspan. The deformable block is subdivided into diagonally opposed triangular zones. In order to save on computation time and to examine a large number of beams coarse meshed numerical models are prepared with zone width along the thickness of the beam equal to $t/10$. Zone width along the span of the beam varies in each model so that the element height to width ratio remains constant and equal to $2:1$. The basic UDEC model described is suitably adjusted to include any vertical joints that cross the beam. All patterns include joints at midspan and the abutments (basic model) while joints are
progressively added in between. The ratio of joint spacing \( s_j \) to beam span \( s \) is taken as \( s_j/s = 0.5 \) (no intermediate joint in each half beam), \( 0.25 \) (1 intermediate joint in each half beam) and \( 0.125 \) (3 intermediate joints in each half beam). Any intermediate joint has the same elastic and mechanical properties with the abutment’s joint while the midspan joint has a normal stiffness two times the abutment joint stiffness so the resulting normal displacement at each half beam is half of the total normal displacement at midspan.

In the right hand discontinuity, which represents the middle section of the beam, both vertical slip and lateral separation are permitted. This is achieved by imposing zero friction angle \( \phi \) and cohesion \( c \). In the left hand discontinuity, which represents the beam abutment, only separation is permitted. This is achieved by imposing very large values for the friction angle and for the cohesion, i.e. \( \phi=89^\circ \) and \( c=10 \) GPa, in order to prevent shear slip.

The normal \( k_n \) and shear \( k_s \) stiffness of the discontinuities are varied from very stiff to relatively soft, i.e. \( k_n=k_s=1000 \), 100 and 10 GPa/m. The stiffest value taken is 1000 GPa/m, which corresponds to an unweathered closed joint. The lowest value of 10 GPa/m corresponds e.g. to a 3mm width filled discontinuity with weathered rock or soil like material of 30MPa deformability modulus.

The deformable beam is considered to behave elastically, with a Young’s modulus of intact rock \( E=10 \), 30 and 100 GPa and zero Poisson’s ratio. Loading of the beam is due to its own unit weight \( \gamma \) which is taken equal to 30 kN/m².

In order to find the limiting thickness value for snap-through failure, an iterative procedure is adopted, in which the value of \( t \) is incrementally decreased until a pair of close enough thickness values is established, one rendering the beam stable and the other unstable. Thus, stable/unstable pairs for the parameters \( s_n \) and \( Q_n' \) are determined.

### 4.2 Results

The stable thicknesses of each beam configuration are shown in the graph of Fig. 4 for the three values of the joint stiffness examined. A reduction of the safe slenderness with the reduction of the joint stiffness is observed, indicating the significant effect of the stiffness of the joints to the stability of the beam. In the same diagram the maximum slenderness and the corresponding normalized load for the three stiff joints voussoir beam model calculated by the analytic solution of Sofianos (1996) is shown. It is observed that numerically obtained values for a jointed beam with three stiff joints are more daring than the analytic ones. This is attributed to the larger stiffness of any discretized continuous structure as are the coarse meshed models used. Models with finer meshes are required in order to define more accurately the stable/unstable pairs of the normalized load and slenderness of the multi-jointed beams.

![Graph showing the relationship between normalized load and span to thickness ratio](image_url)

**Fig. 4.** \( s_nQ_n' \) pairs at incipient buckling for the multiple jointed beams analyzed numerically.
5. DISCUSSION

Analytical solutions provided by Sofianos (1996) may be used to evaluate the stability of a voussoir beam with three stiff joints at the abutment and at midspan. Three modes of failure may be examined: crushing of the rock at the abutments or midspan, shear sliding at the abutments and snap through failure of the beam.

For multiple jointed roof rock beams with compliant joints no analytic solution exists. Numerical simulations with a distinct element code provide a potential for assessing the mechanical behaviour of such beams. For preliminary design purposes numerical results obtained by Nomikos et al. (2002) may be used for multiple jointed roof beams with geometries and mechanical properties pertinent to those analyzed. Normalized contact length at the abutments and at midspan of the beam as well as beam deflection at midspan may be obtained from the graphs provided. The other parameters of the mechanical behaviour of the beam may be evaluated from simple formulae. Thus, jointed beams loaded with their own weight only and span to thickness ratios that does not cause large deflections may be checked against crushing of the rock at the critical sections and shear sliding at the abutments.

A preliminary chart that correlates the slenderness of the beam with the normalized load at incipient buckling, was obtained herein from coarse meshed numerical models. This chart may be used for initial checking the stability of the jointed beam against snap through failure, for beams with geometrical and mechanical properties similar to those examined.

REFERENCES